

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS
METHODS
UNITS 1&2

SOLUTIONS

Section Two:
Calculator-assumed

 Student name:

|  |  |
| --- | --- |
| Number of additionalanswer booklets used(if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number ofquestionsavailable | Number ofquestions tobe answered | Workingtime(minutes) | Marksavailable | Percentageofexamination |
| Section One:Calculator-free | 9 | 9 | 50 | 58 | 35 |
| Section Two:Calculator-assumed | 12 | 12 | 100 | 94 | 65 |
|  |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the Year 12 Information Handbook 2021. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 10 (4 marks)

The value of a block of land, in thousands of dollars, years after the start of the year , can be modelled by the equation , where is a positive constant.

At the start of , the land was valued at .

(a) Show that the value of is , when rounded to decimal places. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ writes equationü solves to more than dp (and then rounds) |

(b) Assuming that the model remains valid into the future, during which year the value of the block will first exceed ? (2 marks)

|  |
| --- |
| Solution |
| Hence during the year (accepts at the start of 2036) |
| Specific behaviours |
| ✓ writes and solves equationü states correct year |

Question 11 (5 marks)

Sector subtends an angle of in a circle with centre and radius .

(a) Express as an exact and simplified radian measure. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value |

The area of sector is cm2.

(b) Determine the radius of the circle. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ sets up the equationü calculates radius |

(c) Determine the area of the minor segment bounded by arc and chord . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ writes equationü calculates area |

Question 12 (6 marks)

Let , where is measured in radians.

The graph of is shown.

Two points, and , lie on the curve with
-coordinates and respectively,
where .

The secant through is also shown.

(a) Use the difference quotient to calculate, to decimal places, the slope of secant when

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses correct values in quotient✓ correct value |

(i) . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(ii) . (1 mark)

(b) Complete the following table to determine an estimate, correct to decimal places, for the slope of secant as the value of tends to . (2 marks)

|  |
| --- |
| Solution |
| As . |
| Specific behaviours |
| ✓ calculates both quotients correctly (at least 5 dp)ü correct estimate, to dp |

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

(c) Briefly describe how your answer to part (b) relates to the graph of at the point . (1 mark)

|  |
| --- |
| Solution |
| It is the slope of the tangent to the graph at the point . |
| Specific behaviours |
| ✓ states slope at point |

Question 13 (11 marks)

A function is defined by .

(a) Complete the following table. (2 marks)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

|  |
| --- |
| Solution |
| See table |
| Specific behaviours |
| ✓ü per error |

(b) (i) Use calculus to determine the coordinates of all stationary points of the graph
 . (3 marks)

|  |
| --- |
| Solution |
|  is stationary at and . |
| Specific behaviours |
| ✓ shows ü solves ü states coordinates of both points |

 (ii) Use a calculus method to determine the nature of the stationary points found in (b)(i).
 (2 marks)

|  |
| --- |
| Solution |
|  and It’s a (horizontal) point of inflection at anda minimum point at  |
| Specific behaviours |
| ü uses 1st derivative test or 2nd derivative test correctlyü states the nature of both stationary points |

(c) Sketch the graph of on the axes below for . (4 marks)



|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ locates domain endpointsü passes through (3, 0)ü behaviour at stationary pointsü smooth curve throughout |

Question 14 (9 marks)

Data from repairs to smartphones showed that were Android (A) and the remainder iOS (I). The type of repair was classified as screen (S) or other (O), and of the smartphones that required screen repairs, were Android.

(a) State the values of and (2 marks)

495

340

265

A

S

|  |
| --- |
| Solution |
|  and  |
| Specific behaviours |
| ✓ two correct valuesü all correct |

(b) Determine the probability that a randomly selected smartphone from those repaired

(i) was an iOS smartphone. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct probability |

(ii) required a screen repair or was an Android smartphone. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates numeratorü correct probability |

(iii) was an iOS smartphone given that it required a screen repair. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct numeratorü correct denominator |

(c) Use two of the above probabilities to explain whether the repair data indicates possible independence of type of smartphone and type of repair. (2 marks)

|  |
| --- |
| Solution |
| Independence appears unlikely since is not close to . |
| Specific behaviours |
| ✓ states independence unlikelyü justifies by comparing relevant probabilities |

Question 15 (8 marks)

Two events and are such that and .

Determine the following probabilities.

(a) when and are mutually exclusive. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates ü correct probability |

(b) when . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates suitable methodü correct probability |

(c) when and are independent. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates or uses suitable methodü correct probability |

(d) when . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates ü correct probability |

Question 16 (7 marks)

An aeroplane takes off from an airport situated at an altitude of metres above sea level and climbs metres during the first minute of flight. In each subsequent minute, its rate of climb reduces by .

(a) Determine the **increase in altitude** of the aeroplane during the sixth minute. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses ü correct increase |

(b) Deduce an explicit rule in simplified form for the **altitude** of the aeroplane at the end of the minute. (3 marks)

|  |
| --- |
| Solution |
|  will be sum of terms plus initial altitude: |
| Specific behaviours |
| ✓ correct use of sum formulaü includes initial altitudeü simplifies (to last or second last line) |

(c) Determine the altitude of the aeroplane after minutes. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates correct term |

(d) Determine the maximum altitude the aeroplane will reach. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct altitude |

Question 17 (10 marks)

Particle P is moving along the -axis so that its displacement, in cm, at time seconds, , is given by .

(a) Sketch the displacement-time graph of particle P on the axes below. (3 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ü turning pointü both interceptsü parabolic shape |



(b) Use a calculus method to determine the velocity of particle P at the instant it reaches the origin. (3 marks)

|  |
| --- |
| Solution |
| Reaches origin when . |
| Specific behaviours |
| ✓ indicates correct timeü obtains velocity functionü correct velocity |

(c) Particle Q is also moving along the -axis, but with a constant velocity. When , it has the same displacement and velocity as particle P. Determine when particle Q reaches the origin. (4 marks)

|  |
| --- |
| Solution |
| Displacement equation (tangent to curve at ):Reaches origin:Hence Q reaches origin when seconds. |
| Specific behaviours |
| ✓ displacement and velocity when ü displacement equationü equates displacement to ü solves for correct time |

Question 18 (8 marks)

A random selection of spanners is made from a collection of different spanners,
of which are metric and the remainder imperial.

(a) Determine the probability that the selection contains all imperial spanners. (3 marks)

|  |
| --- |
| Solution |
| Total possible selections is .Number of imperial spanners is .Ways to select all imperial is . |
| Specific behaviours |
| ✓ calculates number of all possible selectionsü calculates number of ways to select all imperialü finds probability |

(b) Determine the probability that the selection contains

(i) all metric spanners. (2 marks)

|  |
| --- |
| Solution |
| Ways to select all metric is . |
| Specific behaviours |
| ✓ calculates number of ways to select all metric✓ correct probability |

(ii) at least one imperial spanner. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct probability |

(iii) at least one metric spanner and at least one imperial spanner. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ probability all of same typeü correct probability |

Question 19 (7 marks)

A length of wire cm long is cut into two pieces.
One piece is bent into a right triangle with sides
of length and cm and the other piece is
bent into a square of side cm.

(a) Show that (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ equation relating and  |

(b) Formulate an expression for the combined area of the triangle and square in terms of and hence use calculus to determine the minimum value of this total area.

 Justification of the minimum nature is not required. (6 marks)

|  |
| --- |
| Solution |
| The minimum total area is .  |
| Specific behaviours |
| ü total area in terms of ü total area in terms of (expanded and simplified)ü derivativeü equates derivative to ü optimum value of ü calculates and states minimum area |

Question 20 (10 marks)

Three small weights and , each attached to a spring, are oscillating vertically above level ground. The height, cm, above the ground of each weight at time seconds, is given by

(a) State which two weights are oscillating with the same amplitude, and state what this common amplitude is. (2 marks)

|  |
| --- |
| Solution |
| Weights and - their amplitude is cm. |
| Specific behaviours |
| ✓ correct weightsü correct amplitude |

(b) State which two weights are oscillating with the same period, and state what this common period is. (2 marks)

|  |
| --- |
| Solution |
| Weights and - their period is s. |
| Specific behaviours |
| ✓ correct weightsü correct period |

(c) State which of the weights reaches furthest above the ground and state the time at which it first reaches this position. (3 marks)

|  |
| --- |
| Solution |
| Hence weight reach*es* furthest above the ground.When:This first occurs when s. |
| Specific behaviours |
| ✓ states correct weightü equates trig function to ü solves for correct time |

(d) Determine the length of time during the first seconds for which . (3 marks)



|  |
| --- |
| Solution |
| Use CAS to graph heights and identify required interval.Length of time: |
| Specific behaviours |
| ✓ indicates one endpointü indicates second endpointü calculates difference (accepts 0.17 s) |

Question 21 (9 marks)

The graph of the hyperbola is shown below, where and are constants.



(a) State the equations of the horizontal and vertical asymptotes of the hyperbola. (2 marks)

|  |
| --- |
| Solution |
| Horizontal: Vertical: . |
| Specific behaviours |
| ✓ equation for horizontal asymptote✓ equation for vertical asymptote |

(b) Determine the value of and the value of . (2 marks)

|  |
| --- |
| Solution |
| From asymptote, .Using : |
| Specific behaviours |
| ✓ value of ü value of  |

(c) Add the line to the graph of the hyperbola and state the number of points of intersection it will have with the hyperbola. (2 marks)

|  |
| --- |
| Solution |
| See graph for line.It will have points of intersection with the hyperbola. |
| Specific behaviours |
| ✓ correct straight line (both intercepts correct)ü correct number of intersections |

(d) The line is tangential to the hyperbola, where is a constant. Use an algebraic method to determine all possible values of . (3 marks)

|  |
| --- |
| Solution |
| Require one solution to intersection of lines:For one solution, quadratic discriminant :Using CAS: . |
| Specific behaviours |
| ✓ obtains quadratic from equating both linesü uses discriminant to form equation in ü both correct values |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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